International Journal of Modern Computation, Information and Communication Technology 2020;3(1):42-47.
ISSN: 2581-5954
http://ijmcict.gjpublications.com

Research Article

Orthogonal Idempotents and Continuous Fredholm Operators under Perturbations

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Abstract

In the present work, authors presented characterization on continuity of Fredholm operators when perturbed by orthogonal idempotents in Banach space. In particular, authors show that these operators are continuous under pertubations in Hilbert spaces.

Keywords: Fredholm operator; Continuity; Perturbation; Orthogonality; Idempotency.

Introduction

Continuous operators have become one of the most important classes of linear operators in Banach spaces and it has appeared as an interesting area of pure mathematics that is concerned with the global and topological properties of systems of differential equations [1-4]. Therefore, many researchers have considered several types of Fredholm operators and perturbation classes of Fredholm operators for research. In [5-6] the authors obtained results on Fredholm operators and its applications while [7] analyzed perturbation classes of Semi-Fredholm and Fredholm operators noted that the closed and densely defined linear operators on Banach spaces are either upper semi-Fredholm operators or lower semi-Fredholm operators. In [8] the researchers studied perturbations of spectral operators and applications to identity of bounded Perturbations.

Further, [9] obtained results on the characterization of unbounded and bounded Fredholm operators, and solved some problems on Fredholm alternative. The study of [10] showed that invertibility preserving maps preserve idempotents in Banach spaces and on the other hand [11] worked on the linear maps preserving the set of Fredholm operators. Moreover, [3] researched extensively on perturbations and Weyl's theorem while [5] thoroughly worked on Weyl's type theorems and perturbations, also, [16] surveyed on Perturbation analysis of generalized inverses of linear operators in Banach spaces and new stability characterization of generalized inverses in Banach spaces applicable in global analysis, and necessary and sufficient conditions that Moore-Penrose is continuous in Hilbert spaces. Therefore, [8-9] carried out a research on the norm of Idempotents in C^*-algebras and range projections of idempotents in $C^{\supset}$ −algebras and [1] researched on the maps preserving Fredholm operators on Hilbert C^*modules.

Moreover, [19] gives characterization of closed Fredholm and semi-Fredholm operators and its perturbations while [10] obtained necessary and sufficient conditions for the continuity of the spectrum and spectral radius functions at a point of a Banach algebra and [1] characterized Hankel and Toeplitz transforms on H^1 by considering Continuity, Compactness and Fredholm Properties.

Authors in [2,5] determined spectrally bounded Jordan derivations on Banach algebras and norm derivation respectively. The algebras generated by mutually orthogonal idempotent operators was studied by [8] and established that finite dimensional spaces and commuting families of idempotents are diagonalizable and generates reflexive algebras. Lastly, the study of the numerical ranges and the spectrum has been fascinating area of study to many mathematicians, for instance [4] described the

Received: 17.04.2020; Received after Revision: 20.05.2020; Accepted: 22.05.2020

closure of the numerical range of the product of two orthogonal projections in Hilbert space, and in a closed convex hull of some explicit parameters of a point in the spectrum. Many of the researchers have characterized Fredholm operators and determined properties for the invertibility of compact operators in Banach spaces but not for the continuity of Fredholm operators when perturbed by orthogonal idempotent in Banach spaces. In this paper we consider continuity of Fredholm operators when perturbed by orthogonal idempotents in Banach spaces. In this research, continuity of Fredholm operators when perturbed by orthogonal idempotents in Banach spaces have been discussed in detail. Subsequently, the following definitions are fundamental to this work.

Preliminaries

Definition 2.1

Let X and Y be normed linear spaces and $T : X \to Y$ be a linear map. Let $\|\cdot\|_X$ and $\|\cdot\|_Y$ denote norm of X and Y respectively, then T is continuous if whenever $f_n \to f$ implies that $T(f_n) \to T(f)$ then $\lim_{n\to\infty} \|f - f_n\|_X = 0 \Rightarrow \lim_{n\to\infty} \|T(f) - T(f_n)\|_Y = 0.$

Definition 2.2

Let T be densely defined closed operator on X. T is said to be Fredholm operator if and only if that: $\alpha(T)$ is finite, $\beta(T)$ is finite and $R(T)$ is closed in X. T is upper semi-Fredholm operator if $\alpha(T)$ is finite, and $R(T)$ is closed in X are satisfied, while it is a lower semi-Fredholm if $\beta(T)$ is finite, and $R(T)$ is closed in X.

Definition 2.3

An operator T is uniformly continuous if for every $\varepsilon > 0$ there is a positive integer M such that $\|x_m - x_n\| < \varepsilon$ whenever $n > m \geq M$.

Definition 2.4

A sequence $(B_n)_{n\in N} \subset B(H)$ is uniformly convergent to $B_\infty \in B(H)$ if $\lim_{n\to\infty} \|B_n - B_\infty\| = 0,$ is strongly convergent to $B_\infty \in B(H)$ if for all $f \in H$ and $\lim_{n\to\infty} \|B_n f - B_\infty f\| = 0,$ or weakly convergent to $B_\infty \in B(H)$ if for all $f, g \in H$ and $\lim_{n\to\infty} \langle B_n f - B_\infty f, g\rangle = 0.$

Definition 2.5

Let $Q \in B(H)$. Then Q is an idempotent if $Q^2 = Q$ is satisfied.

Results and discussion

In this section authors have characterized continuity of Fredholm operators perturbed by orthogonal idempotents in Banach spaces.

Proposition 3.1.

Let $F_{OI}(H)$ be the three dimensional and $J_1, J_2 \in I_O(H)$. Then $T \in F_{OI}(H)$ is continuous and $J_1 \sim J_2$ if and only if there exists $C, D \in \mathcal{A} \subseteq F_{OI}(H)$ such that $C \neq D$, $J_1 \leq C$, $J_1 \leq D$, $J_2 \leq C$ and $J_2 \leq D$.

Proof.

Suppose $J_1 \sim J_2$, then J_1 and J_2 are linear we have $J_1 = x \otimes h$ and $J_2 = x \otimes f$ $\forall$ $x \in \mathcal{A}$ and $h, f \in \mathcal{A}^*$ with $h(x) = f(x) = 1$. Furthermore, if $h \neq f$, then h and f are continuous. Let $T : J_1 \to J_2$ for all $t, w \in \mathcal{A}$ such that $h(t) = h(w) = f(w) = 0$ and $f(t) = 1$. Therefore, $C = x \otimes h + t \otimes (f - h)$ and $D = x \otimes h + (t + w) \otimes (f - h)$ are orthogonal idempotents satisfying $J_1 \leq C$, $J_1 \leq D$, $J_2 \leq C$ and $J_2 \leq D$. Hence $J_1, J_2 \in I_O(H)$ and $T \in F_{OI}(H)$ are continuous at $t, w \in \mathcal{A} \in F_{OI}(H)$.

Proposition 3.2.

Let $F_{OI}(H)$ be two dimensional. Then $T \in F_{OI}(H)$ is continuous if and only if for every $J_1, J_2 \in I_O(H)$ there exists $C, D \in \mathcal{A} \subseteq F_{OI}(H)$ such that $J_1 \sim C$ and $J_1 \sim D$.

Proof.

Let $J_1 = x \otimes h$ and $J_2 = y \otimes f$ then $h(y) \neq 0$ or $f(x) \neq 0$ such that J_1 and J_2 are continuous at $\mathcal{A}$. Since $D \in \mathcal{A} \subseteq F_{OI}(H)$, then $D = \frac{1}{h(y)}(y \otimes h)$ satisfying that $J_1 \sim C$ and $J_1 \sim D$. Suppose $J_1 \perp J_2$, then $h(x) = f(x) = 0$ and since J_1 is a rank one orthogonal idempotent satisfying $J_1 \sim C$. Then, $\mathcal{A} = x \otimes r$ for all $r \in F_{OI}(H)$ with $r(x) = 1$ or there exists $\mathcal{A} = t \otimes h$ for all $t \in F_{OI}(H)$ such that $h(x) = 1$. Therefore, $J_1 \perp J_2$ is continuous for all $x \in J_1$ and $x \in J_2$ respectively. Hence $J_1 \sim C$ and $J_1 \sim D$ are continuous at $T \in F_{OI}(H)$.

Proposition 3.3.

Let $F_{OI}(H)$ be the four dimensional and $J_1 \in \chi \subseteq F_{OI}(H)$, then $T \in F_{OI}(H)$ is continuous if for every $E_1, E_2, E_3 \in F_{OI}(H) \setminus \{0, I\}$ satisfying $J_1 E_1 = E_1 J_1$, $J_1' \neq E_1'$, $E_2 E_3 = E_3 E_2$, $E_2' \neq E_3'$ and $\{J_1 E_1\}' \subset \{E_2 E_3\}'$. Hence, $\{J_1 E_1\}' \subset \{E_2 E_3\}'$.

Proof.

Let $J_1 \in \chi$ then $J_1 = (I - J_1)'$ such that J_1 is continuous at $\chi \in F_{OI}(H)$. Therefore, with respect to the direct sum decomposition of $F_{OI}(H) = \operatorname{Im} J_1 \oplus \ker J_1$ and $J_1 = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$. If $E_1, E_2, E_3 \in \chi \setminus \{0, I\}$, then $J_1 E_1 = E_1 J_1, J_1' \neq E_1'$ and $E_1 = \begin{pmatrix} \lambda & 0 \\ 0 & D \end{pmatrix}$ for all $\lambda \in \{0, I\}$ and $D \in J_1(\ker J_1)$.

Therefore, the direct sum decomposition of $F_{OI}(H) = \operatorname{Im} J_1 \oplus \operatorname{Im} D \oplus \ker D$ for all $J_1, D \in I_O(H)$ $J_1 = \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $E_1 = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Hence, $\{J_1 E_1\}'$ having $H, R \in I_O(H)$, we have the matrix $\begin{pmatrix} \delta & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & R \end{pmatrix}$ for all $\delta \in \{0, I\}$. From $\{J_1 E_1\}' \subset \{E_2 E_3\}'$, we have $E_2 = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 I & 0 \\ 0 & 0 & \mu_3 I \end{pmatrix}$ and $E_3 = \begin{pmatrix} \mu_4 & 0 & 0 \\ 0 & \mu_5 I & 0 \\ 0 & 0 & \mu_6 I \end{pmatrix}$ with $\mu_j = \{0, I\}$, $j = 1, \ldots, 6$. Let $E_2 = I - E_2$, we have $\mu_1 = 0$ and $\mu_4 = 0$. If $\mu_2 \neq \mu_5$, then without the loss of generality we have $\mu_1 = I$ and $\mu_5 = 0$ and it implies that $\mu_6 = I$. Hence $\{J_1 E_1\}' \subset \{E_2 E_3\}'$. Suppose J_1 does not belong to $\chi \in F_{OI}(H)$, then direct sum decomposition of $F_{OI}(H) = H_1 \oplus H_2 \oplus H_3 \oplus H_4$ with $\operatorname{Im} J_1 = H_1 \oplus H_2$ and $\ker J_1 = H_3 \oplus H_4$. We have $J_1 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, such that $E_1, E_2, E_3 \in \chi$ defined by

$$E_1 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

and $E_3 = J_1$. Therefore, $J_1 E_1 = E_1 J_1$, $J_1' \neq E_1'$, $E_2 E_3 = E_3 E_2$, $E_2' \neq E_3'$ and hence $\{J_1 E_1\}' \subset \{E_2 E_3\}'$.

Proposition 3.4.

Let $T \in F_{OI}(H)$ and $J_1, J_2 \in I_O(H)$. Let $T : V_1 \to V_2$ be a continuous map, then it implies that $T : V_1^n \to V_2^n$ is also a continuous map.

Proof.

Let $T : V_1 \to V_2$ be a continuous map of orthogonal idempotents J_1 and J_2. Then, if

$G \in F_{OI}(H)$ such that G can be decomposed into a direct sum $G = G_1 \oplus G_2$ where G_1 is a closed subspace of dimension $n-1$. Therefore, $TG = TG_1 \oplus TG_2$ is closed and continuous, thus TG_1 is also closed and continuous. Hence, every direct sum of a closed subspace with a finite dimensional subspace is closed.

Proposition 3.5.

Let $T_1 T \in F_{OI}(H)$. Then $T_1 : \chi_n(V_1) \to \chi_n(V_2)$ is a continuous map on V_1 and V_2 such that $T : V_1^* \to V_2$ is also a continuous map. If for every $U \in \chi(V_1)$ and $f \in V_1^*$, we have $U^* f = 0$ if and only if $T_1(U)Tf = 0$. Then $T \in F_{OI}(H)$ is compact and $T_1 UT = TU^*T^{-1}$ for all $U \in \chi_n$, and $V_1, V_2 \in I_O(H)$.

Proof.

Let b be a scalar, such that $T(bf) = g(b)Tf$ for all $f \in V_1^*$ and $T^{-1}(by) = g^{-1}(b)T^{-1}y$ for all $y \in V_1^*$. Suppose $U \in \chi(V_1)$ and $TU^*T^{-1} : V_1^* \to V_2$ is continuous with an orthogonal idempotent of the rank n, then $\ker T_1 U = \ker TU^*T^{-1}$ and $\operatorname{Im} T_1 U = \operatorname{Im} TU^*T^{-1}$ such that TU^*T^{-1} and $T_1 U$ are continuous orthogonal idempotents which are equal. For instance, TU^*T^{-1} is bounded and continuous for every $U \in \chi(V_1)$. Moreover, if $y \in \ker T_1 U$ then $y \in Tf$ for all $f \in \ker U^*$ and $f \in \ker U^* T_1 = \ker TU^*T_1$ such that $\ker T_1 U = \ker TU^*T_1$ for all $U \in \chi_n(V_1)$. Suppose n co-dimension exists; for every continuous closed subspace of $z \in V_2$ we have $U \in \chi(V_1)$ such that $z = \ker T_1 U$ for all $\ker T_1 U = \ker TU^*T_1 = T(\ker U^*)$ and $T_1(z) = T_1(\ker T_1 U) = \ker U^*$ is compact. Since T and T_1 are continuous operators, there exists a sequence $y_r \subset V_2$ with $y_r \to 0$ and $T_1 y_r \to \lambda \neq 0$ as $r \to \infty$, with $\lambda(x_i) \neq 0$, $x_i, \cdots x_n \in V_1$ and $x_i, \ldots, x_n$ are continuous. Similarly, if $f_i, \ldots, f_n \in V_1^*$ and $f_i(x_j)$, $1 \le i, j \le n$ then $f_i, \ldots, f_n$ and $Tf_i, \ldots, Tf_n$ are continuous and there exists $c \in V_2^*$ such that $c(Tf_1) = 1$ and $c(Tf_2) = \ldots = c(Tf_2) = 0$.

Theorem 3.6.

For a positive integer $J_1, J_2 \in I_O(H)$ such that $J_1, J_2 \in \chi \subseteq F_{OI}(H)$ and $J_1 \neq J_2$ satisfying the following:

(i.) $rank(J_1 - J_2) = 1$ and either $\operatorname{Im} J_1 = \operatorname{Im} J_2$ or $\ker J_1 = \ker J_2$.

(ii.) There exists $x \in \operatorname{Im} J_1$ and f continuous at $f \in (F_{OI}(H))^*$ such that $f(\operatorname{Im} J_1) = \{0\}$, $J_2 = J_1 + x \otimes f$ or a nonzero $x \in \ker J_1$ and $f \in (F_{OI}(H))^*$, $\ker J_1 = \{0\}$ and $J_2 = J_1 + x \otimes f$.

(iii.) For $C, D \in \chi \subseteq F_{OI}(H)$, satisfying $C \neq D$ and $\{J_1, J_2\}^{\perp} \subset \{C, D\}^{\perp}$.

Proof.

Case (i.): Suppose $J_1 \sim J_2$, and $J_1 \neq J_2$ such that $rank(J_1 - J_2) = 1$. Thank, let $J_1 = \sum_{q=1}^{n} t_q \otimes h_q$, $t_1, \ldots, t_n \in H$ and $h_1, \ldots, h_n \in (H)^*$ are continuous at $F_{OI}(H)$, then $J_2 = \sum_{q=1}^{n} t_q \otimes h_q + k \otimes s$ for $k \in H$ and $s \in H^*$. If both operators $\{k, t_1, \ldots, t_n\}$ and $\{s, h_1, \ldots, h_n\}$ are continuous at J_2 yields $\operatorname{Im} J_1 \subset \operatorname{Im} J_2$ then they are continuous orthogonal idempotents. If $\operatorname{Im} J_1 = \operatorname{Im} J_2$ and $\ker J_1 = \ker J_2$ or $h \in span\{h_1, \ldots, h_2\}$ we obtain $\ker J_1 = \ker J_2$. In addition, let $x \in J_1$ and $f \in F_{OI}(H)$ such that $\omega = x \otimes f$ if for $J_1 t + \omega t \in \operatorname{Im} J_1$ and since $J_1 + x \otimes f$ is an orthogonal idempotent and continuous. Therefore, $f(x) = 0$ and $f(\operatorname{Im} J_1) = \{0\}$ from $(J_1 + x \otimes f)^2 = J_1 + x \otimes f$.

Case (ii.) Suppose $J_2 = J_1 + x \otimes f$ for $x \in \operatorname{Im} J_1$ and $f \in (F_{OI}(H))^*$ satisfying $f(\operatorname{Im} J_1) = \{0\}$ and that $C, D \in \chi \subseteq F_{OI}(H)$, satisfying $C \neq D$ and $\{J_1, J_2\}^\perp \subset \{C, D\}^\perp$ are orthogonal idempotents. Therefore, $\{J_1, J_2\}^\perp = T \in \chi \subseteq F_{OI}(H) : \operatorname{Im} J_1 \subset \ker T$ and $\operatorname{Im} T \subset \ker J_1 \mathrm{I} \ker f$. Assuming that $\operatorname{Im} C \subsetneq \operatorname{Im} J_1$, then $t \in \operatorname{Im} C$ while s and t not in $\operatorname{Im} J_1$ we have $t = t_1 + t_2$ with $t_1 \in \operatorname{Im} J_1$, $t_2 \in \ker J_1$ and $t = 0$. If $y_1, \ldots, y_n \in \ker J_1 \mathrm{I} \ker f$ such that $y_1, \ldots, y_n = 0$, there exists $f_1, \ldots, f_n \in F_{OI}(H)$ for all $f_r(\operatorname{Im} J_1) = \{0\}, r = 1, \ldots, n$ and $f_1(t_2) = f_1(t) \neq 0$. Then $\omega = \sum_{r=1}^{n} \mu_t \otimes f_r \in \{J_1, J_2\}^\perp$ and $\operatorname{Im} C \subset \operatorname{Im} J_1$ are orthogonal idempotents of the rank n thus, $\operatorname{Im} C = \operatorname{Im} J_1$ and $\operatorname{Im} D = \operatorname{Im} J_1$ also $\ker J_1 \mathrm{I} \ker f \subset \ker C$ and $\ker J_1 \mathrm{I} \ker f \subset \ker D$.

Case (iii.) Now if $C \neq D$ and $\operatorname{Im} C = \operatorname{Im} D$ it follows that $\ker C \neq \ker D$ such that $\ker C \mathrm{I} \ker D$ is of co-dimension at least $n+1$ since $\ker J_1 \mathrm{I} \ker f \subset \ker C \mathrm{I} \ker D$. It implies that $\ker J_1 \mathrm{I} \ker f = \ker C \mathrm{I} \ker D$, thus $\{J_1, J_2\}^\perp$ is satisfying if for some $x \in \ker J_1$ and $f \in (F_{OI}(H))^*$, we have $f \in \ker J_1 = \{0\}$ and $J_2 = J_1 + x \otimes f$. Since J_1 and J_2 are orthogonal idempotents and $J_1 \neq J_2$, we have $\ker J_1 \neq \ker J_2$ or $\operatorname{Im} J_1 \neq \operatorname{Im} J_2$. If $\ker J_1 \neq \ker J_2$ then $\ker J_1 \otimes \ker J_2$ and J_1 is continuous with finite codimension of $\ker J_1 \mathrm{I} \ker J_2$ and if $P \in F_{OI}(H)$ be a finite dimensional then $\ker J_1 = (\ker J_1 \mathrm{I} \ker J_2) \oplus P$ such that $F_{OI}(H) = \operatorname{Im} J_1 \oplus (\ker J_1 \mathrm{I} \ker J_2) \oplus P$. Furthermore, $(F_{OI}(H))^*$ satisfying $f(\operatorname{Im} J_1 \oplus (\ker J_1 \mathrm{I} \ker J_2)) = \{0\}$ and $x \in \operatorname{Im} J_1$. If $C = J_1$ and $D = J_1 + x \otimes f$, then D is an orthogonal idempotent of rank n. Therefore, if $T \in \{J_1, J_2\}^\perp$ we have $\operatorname{Im} J_1 \subset \ker T$ with $\operatorname{Im} J_1 = \operatorname{Im} J_2$ such that $TD = 0$ and $\operatorname{Im} T \subset \ker J_1 \mathrm{I} \ker J_2$ implies that $f(\operatorname{Im} T) = 0$ and $TD = 0$. Since $\{J_1, J_2\}^\perp = \{C, D\}^\perp$ it follows that $\ker J_1 \mathrm{I} \ker f \subset \ker J_1$, and if for all $a \in F_{OI}(H)$ we have $\operatorname{Im} J_1 \oplus (\ker J_1 \mathrm{I} \ker f) \oplus span\{a\}$ and $\{C, D\}^\perp = \{T \in \chi \subseteq F_{OI}(H) : T(\operatorname{Im} J_1) = \{0\}, \operatorname{Im} T \subset \ker J_1 \mathrm{I} \ker f\}$.

Therefore, $J_2 \subseteq \ker J_1 \mathrm{I} \ker f$ is orthogonal idempotent operator, hence $J_2 T \neq 0$ for all $T \in \{C, D\}^\perp$. Hence, J_1 and J_2 are continuous identities on $\operatorname{Im} J_1 = \operatorname{Im} J_2$ and both orthogonal idempotents on $\ker J_1 \mathrm{I} \ker f$, thus the space $\operatorname{Im} J_1 \oplus (\ker J_1 \mathrm{I} \ker f)$ is continuous with a codimension one in $F_{OI}(H)$. Therefore, it follows that $J_1 \neq J_2$, then we have $rank(J_1 - J_2) = 1$.

Corollary 3.7.

Let $T \in F_{OI}(H)$ and $\chi \subseteq F_{OI}(H)$ then J_1 and J_3 are orthogonal idempotents continuous at χ such that $J_1 \sim J_3$ for all $J_1, J_3 \in \chi \subseteq F_{OI}(H)$.

Proof.

Let $y = \chi \setminus \{J_1\} = \{J_3 - J_4 : J_3, J_4 \in \chi\}$ then every member of y has a rank of one, for instance $A_1, A_2 \in y$ then $rank(A_1 - A_2) \leq 1$. Similarly, $x \otimes f - g \otimes h$ is of rank one if for x

and g are continuous at f and g respectively. From Theorem 3.6, if $D \in F_{OI}(H)$ is of rank one then we have $rank(A_1 - A_2) \leq 1$ for all $A_1 - A_2 \in D$. Moreover, if $x \in (F_{OI}(H))^* = \{x \otimes f : f \in (F_{OI}(H))^*\}$ for all $x \in F_{OI}(H)$ we have $F_{OI}(H) \otimes f = \{x \otimes f : x \in (F_{OI}(H))^*\}$ for all $f \in (F_{OI}(H))^*$. Therefore, y is continuous at $x \in F_{OI}(H)$ and $y \subset x \otimes (F_{OI}(H))^*$. Hence J_1 and J_3 are orthogonal idempotents and $J_1 \sim J_3$ are continuous at $\chi \in F_{OI}(H)$.

Conclusions

In conclusion, we have obtained results on continuity of Fredholm operators when perturbed by orthogonal idempotents in Banach space for the class of $F_{OI}(H)$.

Conflicts of interest

Authors declare no conflict of interest.

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