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Research Article

Notions of Connectedness in Fuzzy Soft Topological Spaces

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Abstract

Recently, many scholars have studied properties and applications on the soft set theory. In this paper, we study new notions of separate sets, disconnected sets and connected sets called γ-τ-separate set, γ-τ-disconnected sets and γ-τ-connected set. Moreover we give some relationships existing amongst these sets in fuzzy soft topological spaces.

Keywords: Fuzzy soft topological space, γ-τ-separate set, γ-τ-disconnected set, γ-τ-connected set.

Introduction

Fuzzy sets were introduced over many decades back and this is a paradigm shift that first gained acceptance by mathematicians in the Far East and its successful application has ensured its adoption around the world [1-3]). Fuzzy sets are an extension of classical set theory and are used in fuzzy logic as seen in [4,5]. In classical set theory the membership of elements in relation to a set is assessed in binary terms according to a crisp condition, that is, an element either belongs or does not belong to the set [6,7].

By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in relation to a set whereby this is described with the aid of a membership function valued in the real unit interval (0, 1). Fuzzy sets are an extension of classical set theory since, for a certain universe, a membership function may act as an indicator function, mapping all elements to either 1 or 0, as in the classical notion [9]. Specifically, a fuzzy set is any set that allows its members to have different grades of membership (membership function) in the interval (0, 1). A fuzzy set on a classical set X is defined as $\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$.

The membership function $\mu_A(x)$ quantifies the grade of membership of the elements x to the fundamental set X. An element mapping to the value 0 means that the member is not included in the given set, 1 describes a fully included member. Values strictly between 0 and 1 characterize the fuzzy members. In [10] they introduced the concept of an operation γ on a topological space based on the idea of the α-operation as defined by [11] and consequently he introduced γ-open sets. Several research papers published in recent years using γ-operator due to [12-17].

The notion of γ-open sets (originally called γ-sets) in topological spaces was introduced by [18].The generalization of open and closed set as like γ-open and γ-closed sets was introduced in [19] which is nearly to open and closed set respectively. These notion are plays significant role in general topology [20]. Throughout this paper (X, τ) and (X, τ_γ) will always be fuzzy soft topological spaces. For a subset A of fuzzy soft topological space X, Cl_γ(A), Cl(A), int(A), int_γ denote the γ-closure, closure, interior and γ-interior of A respectively and G_γ is the γ-open set for topology τ_γ on X. In this study, we give necessary and sufficient conditions for separate, disconnectedness and connectedness in fuzzy soft topological spaces.

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Preliminaries

Definition 2.1

Let (X, τ) be a fuzzy soft topological space. A subset A of (X, τ) is said to be γ-open if A⊆*Cl* (*Int*(A))∪(*Int*(*Cl*(A)). The family τ_γ of γ-open sets of (X, τ) is a topology on X which is finer than τ.

Definition 2.2

Let (X, τ) and (X, τ_γ) be fuzzy soft topological spaces. Then the subsets A and B of (X, τ) are said to be γ-τ-separate sets if and only if A and B are non-empty sets, A⋂Cl_γ(B) = ϕ and B⋂Cl_γ(A) = ϕ.

Definition 2.3

Let (X, τ) and (X, τ_γ) be fuzzy soft topological spaces. Then the subsets A of X is said to γ-τ-disconnected if there exists G_γ and H_γ in τ_γ such that A⋂G_γ and A⋂H_γ≠∅ (A⋂G_γ)⋂(A⋂H_γ)= ∅, (A⋂G_γ)∪(A⋂H_γ)= A. (X, τ_γ) is said to be γ-τ-disconnected if there exists non-empty G_γ and H_γ in τ_γ such that G_γ⋂H_γ= ∅ and G_γ∪H_γ= X .

Definition: 2.4

Let (X, τ) be a fuzzy soft topological space and A be non-empty subset of X. Let G_γ be arbitrary in τ_γ then collection τ_γA = {G_γ⋂A: $G_\gamma \epsilon \tau_\gamma$} is a topology on A called the subspace topology or relative topology of topology τ_γ.

Results and discussions

Theorem 3.1

Let (X, τ) and (X, τ_γ) be fuzzy soft topological spaces, then (X, τ) is γ-τ-disconnected if and only if there exists non-empty proper subset of X which is both γ-open and γ-closed.

Proof 3.1

*Necessity***:** Let (X, τ) be γ-τ-disconnected. Then by definition of γ-τ-disconnected, there exists non-empty set G_γ and H_γ in τ_γ such that G_γ⋂H_γ= ∅ and G_γ∪H_γ=X. Since G_γ⋂H_γ=∅ and H_γ is open in τ_γ. We show that G_γ= X\H_γ, but it is γ-closed. Hence G_γ is non-empty proper subset of X which is γ-closed as well γ-open.

*Sufficiency***:** Suppose A is non-empty proper subset of X such that it is γ-open as well γ-closed. Now A is non-empty γ-closed show that X\A is non-empty and γ-open. Suppose B=X\A, then A∪B=X and A⋂B=∅. Thus, A and B are non-empty disjoint γ-open as well as γ-closed subset of X such that A∪B=X. Consequently, X is γ-τ-disconnected.

Theorem 3.2

Every (X, τ) discrete space is γ-τ-disconnected if the space contains more than one element.

Proof 3.2

Let (X, τ) be a discrete fuzzy soft topological space such that X={a, b} contains more than one element. But τ is discrete topology so τ={∅, X, {a}, {b}} and the family of all γ-open sets is τ_γ ={∅, X,{a},{b}}. All γ-closed sets are ∅, X, {a}, {b}. Since {a} is non-empty proper subset of X which is both γ-open and γ-closed in X. Finally, we can say that (X, τ) is γ-τ-disconnected by using above theorem.

Theorem 3.3

If (X, τ) a disconnected fuzzy soft topological space and (X, τ_γ) is a fuzzy soft topological space, then (X, τ_γ) is γ-τ-disconnected.

Proof 3.3

As (X, τ) disconnected and τ_γ is finer than τ, then by definition of γ-open and γ-closed set τ_γ⊒τ, τ *is a* fuzzy soft topological $subspace\ of$ τ_γ. Since, is disconnected, τ_γ is also disconnected.

Theorem 3.4

Let A be a non-empty subset of fuzzy soft topological space (X, τ). Let τ_γA be the relative topology on A, then A is γ-τ-disconnected if and only if A is γ-τ_γA-disconnected.

Proof 3.4

Necessity: Let A be a γ-τ-disconnected and let G_γ∪$H\gamma$ be a γ-τ-disconnected on A then by definition of γ-τ-disconnected, there exists non-empty G_γ and H_γ in τ_γ such that:
A⋂G_γ≠∅, A⋂H_γ≠∅ ------------------- (1)
(A⋂G_γ)⋂(A⋂H_γ) =∅ --------------- (2)
(A⋂G_γ)∪(A⋂H_γ) = A --------------- (3)
Now by the definition of relative topology, if G_γ and H_γ are in τ_γ, then there exists ${G_\gamma}^*$and ${H_\gamma}^*$in

τ_γA such that G_γ*=A $\cap G_\gamma$ and H_γ* = A$\cap H_\gamma$. Now by (1) G_γ* and H_γ* are non-empty. Hence, we have A$\cap G_\gamma$* and A$\cap H_\gamma$*non-empty. Similarly, by (2) and (3), we can say that the intersection (A$\cap G_\gamma$)$\cap$(A$\cap H_\gamma$) =∅ and (A$\cap G_\gamma$)∪(A $\cap H_\gamma$)=A. Consequently, A is γ-τ_γA-disconnected.

Sufficient: Suppose that A is γ-τ_γA-disconnected and M_γA$\cap N_\gamma$A is a γ-τ_γA-disconnection on A. By definition, we can say that

M_γA ≠∅, A ≠∅ -------------------------(4)
M_γA, Aϵ τ_γA ----------------------------(5)
(A$\cap M_\gamma$A)$\cap$(A$\cap N_\gamma$A) =∅--------------(6)
(A$\cap M_\gamma$A)∪(A$\cap N_\gamma$A)=A--------------(7)
Now (5) implies that there exists M_γ*, N_γ*ϵ τ_γ such that M_γA = A$\cap M_\gamma$*, N_γA = A$\cap N_\gamma$*. But by (4) we can say that A$\cap M_\gamma$*, A$\cap N_\gamma$* ≠∅. Now, (A$\cap M_\gamma$A) =A$\cap$(A$\cap M_\gamma$*)=(A$\cap$A)$\cap M_\gamma$*=A$\cap M_\gamma$*. (A$\cap N_\gamma$A)=A$\cap$(A$\cap N_\gamma$*)=(A$\cap$A)$\cap N_\gamma$*= A$\cap N_\gamma$*. Now (6) implies that (A$\cap M_\gamma$*)$\cap$(A$\cap N_\gamma$*) =∅. Now (7) implies that (A$\cap M_\gamma$*)∪(A$\cap N_\gamma$*)=A. Finally, we can say that A is γ-τ-disconnected.

Theorem 3.5

The union of two non-empty γ-τ-separate subsets of fuzzy soft topological space (X, τ) is γ-τ-disconnected.

Proof 3.5

Let A and B be γ-τ-separate subsets of (X, τ). Then by definition of γ-τ-separate sets, we say that A and B non-empty. Let A$\cap$B=∅, B$\cap cl$(A) =∅. Let X\\$cl$(A) =$G_\gamma$ and X\\$cl_\gamma$(B)= H_γ. Then cl_γ (A) and cl_γ (B) are non-empty γ-closed subsets of X which shows that G_γ and H_γ are non-empty γ-open subsets of X. Since, G_γ∪H_γ= (X\\$cl_\gamma$(A))∪(X\\$cl_\gamma$ (B))=X\\$cl_\gamma$(A)$\cap cl_\gamma$(B), we have (A∪B) $\cap G_\gamma$=(A∪B)$\cap$(X\\$cl_\gamma$(A))=[A$\cap$(X \\$cl_\gamma$(A)]∪[B$\cap$(X\\$cl_\gamma$(A)]=∅∪B(A∪B)$\cap G_\gamma$=B.

Similarly, (A∪B) $\cap H_\gamma$=(A∪B)$\cap$(X\\$cl_\gamma$ (B))=[A$\cap$(X\\$cl_\gamma$(B)]∪[B$\cap$(X\\$cl_\gamma$(B)]=∅∪A(A∪B)$\cap H_\gamma$ =A. Now (1) shows that (A∪B)$\cap G_\gamma$≠∅, (A∪B)$\cap H_\gamma$≠∅. Additionally, (2) and (3) shows that [(A∪B)$\cap H_\gamma$]$\cap$[(A∪B) $\cap G_\gamma$]=∅ and [(A∪B)$\cap H_\gamma$]∪[(A∪B)$\cap G_\gamma$]=A∪B. Finally, we can say that there exists G_γ and H_γ in τ_γ such that (A∪B)$\cap G_\gamma$≠∅, (A∪B)$\cap H_\gamma$≠∅, [(A∪B)$\cap H_\gamma$]$\cap$[(A∪B)$\cap G_\gamma$]=∅ and [(A∪B)$\cap H_\gamma$]∪[(A∪ B)$\cap G_\gamma$]=A∪B. So, G_γ∪H_γ is a γ-τ-disconnection of A∪B. Hence, A∪B is γ-τ-disconnected.

Theorem 3.6

Let (X, τ) and (X, τ_γ) be fuzzy soft topological spaces, A be a subset of X and let G_γ∪H_γ be a γ-τ-disconnection of A. Then A$\cap G_\gamma$ and A$\cap H_\gamma$ are γ-τ-separate subsets of fuzzy soft topological space (X, τ).

Proof 3.6

Let G_γ∪H_γ be a given γ-τ-disconnection of subset A of (X, τ). To prove: A$\cap G_\gamma$ and A$\cap H_\gamma$ are γ-τ-separate subsets. We must show that
A$\cap G_\gamma$ and A$\cap H_\gamma$ are non-empty---------------(8)
[cl(A$\cap G_\gamma$)]$\cap$(A$\cap H_\gamma$)=∅------------------------(9)
[cl(A$\cap H_\gamma$)]$\cap$(A$\cap G_\gamma$)=∅-----------------------(10)
Now by our assumption and definition, we can say that there exists $G_\gamma \epsilon$ τ_γ such that
A$\cap G_\gamma$ and A$\cap H_\gamma$ are non-empty--------------(11)
(A$\cap G_\gamma$)$\cap$(A$\cap H_\gamma$)=∅-----------------------------(12)
(A$\cap G_\gamma$)∪(A$\cap H_\gamma$)=A ----------------------------(13)
Evidently, (11) ⇒ (9).

To prove (10) we suppose that it is not possible. i.e.cl(A$\cap G_\gamma$)$\cap$(A $\cap H_\gamma$)≠∅. Then, there exists x ϵ (A$\cap G_\gamma$)$\cap$(A$\cap H_\gamma$) which implies that xϵcl_γ(A$\cap G_\gamma$) and x ϵ A, x ϵH_γ,that is (A$\cap G_\gamma$)$\cap H_\gamma$≠∅. Therefore, (A$\cap G_\gamma$)$\cap$(A $\cap H_\gamma$)≠∅. But it is contrary to (12). Hence, our assumption is wrong which completes the proof.

Theorem 3.7

A subset Y of a fuzzy soft topological space X is γ-τ-disconnected if and only if it is union of two γ-τ-separate sets.

Proof 3.7

Necessity -Suppose Y=A∪B, where A and B are γ-τ-separate sets of X. By Theorem 3.6, A∪B is γ-τ-disconnected. Hence, Y is γ-τ-disconnected.

Sufficiency: Let Y be γ-τ-disconnected. To prove that there exists two γ-τ-separate subsets of A, B in X such that Y=A∪B. By assumption, Y is γ-τ-disconnected show that there exists a γ-τ-disconnection G_γ∪H_γ of Y. Therefore, by definition of γ-τ-disconnected, we can say that there exists G_γ and H_γ in τ_γ such that

Y$\cap G_\gamma$ and Y$\cap H_\gamma$ are non-empty -----------(14)
(Y$\cap G_\gamma$)$\cap$(Y $\cap H_\gamma$)=∅-------------------------(15)
(Y$\cap G_\gamma$)∪(Y$\cap H_\gamma$)=Y-------------------------(16)

Since Y∩$G\gamma$ and Y∩$H\gamma$ in (14) are separated sets then we have A= (Y ∩$G\gamma$) and B=(Y ∩$H\gamma$), then by (15) Y=A∪B. Finally, we can say that there exists two γ-τ-separate subsets A and B in X such that Y=A∪B.

Theorem 3.8

If Y is a γ-τ-connected subset of fuzzy soft topological space X such that Y⊂A∪B, where A and B is γ-τ-connected, then Y⊂A and Y⊂B.

Proof 3.8

Since the inclusion Y⊂A∪B holds by the hypothesis we have (A∪B)∩Y=Y which yields that Y=(Y∩A)∪(Y∩B). Now we want to prove that (Y∩A)=∅, (Y∩B)=∅. Suppose, (Y∩A) ≠∅, (Y∩B)≠∅. Now, (Y∩A)∩cl_γ(Y∩B)⊂(Y∩A)∩(cl_γ(Y)∩(B))=(Y∩(cl_γ(Y))∩(A∩(cl_γ(B))=(Y∩(A ∩(cl_γ(B))=Y∩∅=∅, (Y∩A)∩cl_γ(Y∩B)=∅. Similarly, we can prove that cl(Y∩A)∩(Y∩B)=∅. Hence, from the above result we can say that Y is a union of two γ-τ-separate sets (Y∩A) and (Y∩B). Consequently, Y is γ-τ-disconnected. But this contradicts the fact that Y is γ-τ-connected. Hence we can say that (Y∩A) =∅, (Y ∩ B)=∅. Now, if (Y∩A) = ∅, then Y=∅∪(Y∩B)=(Y ∩ B) which gives that Y⊂B. Similarly, Y⊂A if (Y∩A)=∅.

Conclusions

In the present paper, we have extended the notion of connectedness of fuzzy soft topological spaces to generalized fuzzy soft topological spaces. We have introduced different notions of generalized called γ-τ-separate set, γ-τ-disconnected sets and γ-τ-connected set and studied the relationship between them.

Conflicts of interest

Authors declare no conflict of interest.

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