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Research Article

Characterization of Numbers using Methods of Staircase and Modified Detachment of Coefficients

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Abstract

The present work deals with theory of numbers in which detached coefficients method and staircase method were employed. The two methods were computed with their algorithms and illustrative example to ascertain the results for square of numbers and cubic of integer's numbers.

Keywords: Numbers; Algorithms; Modified Detached Coefficients Method; Staircase Method.

Introduction

Studies on number theory have interested many researchers and mathematicians of the highest class of old that have contributed original work to the number theory that cannot be forgotten [1]. China discovered a magic square as far back as 200BC, the magic square were subsequently introduced into India, Japan and later to Europe. Also, in 1770 Euler published his Anleitung Zur Algebra in two volumes. A French translation, with numerous and valuable additions by Lagrange, was brought out in 1794, and a treatise on arithmetic by Euler was appended to it. The first volume treats of determinate algebra [2]. This contains one of the earliest attempts to place the fundamental processes on a scientific basis: the same had attracted D'Alembert's attention. This work also includes the proof of the binomial theorem for unrestricted real index which is still known by Euler's name, the proof is founded on the principle of the permanence of equivalent forms, but Euler made no attempt to investigate the convergence of the series: that he should have omitted this essential step is the more curious as he had himself recognized the necessity of considering the convergence of infinite series: Vandermonde's proof given in 1764 suffers from the same defect [3].

The second volume of the algebra treats of indeterminate or Diophantine algebra. This contains the solutions of some of the problems proposed by Fermat, and which hitherto remained unsolved [4]. Moreover, Pascal employed his arithmetical triangle in 1653, but no account of this method was printed [5]. The numbers in each line are now called figurate numbers [6]. Those in the first line are called numbers of the first order, those in the second line, natural numbers or numbers of the second order, those in the third line, numbers of the third, and so on. Pascal's arithmetical triangle [8], to any required order, is got by drawing a diagonal downwards from right to the left of the triangle [9]. The objectives of this paper are: Continuation on the computation of squares and cubes of integers up to five digits using the modified detached coefficients method and the Staircase method and to determine which of the methods is faster in terms of computation..

Research Methodology

Consider the expansion of $(x_1+2x_2+x_3)^3$.This can be written using the multinomial theorem as

$$(x_1+2x_2+x_3)^3 = \binom{3}{n_1,n_2,\ldots,n_k} x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k}$$

$$=\binom{3}{0,0,0} x_1^3 + \binom{3}{2,1,0} x_1^2 + \binom{3}{2,0,1} 3x_1^2x_3 + \binom{3}{1,2,0} x_1(2x_2)2x_3 +$$

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$\binom{3}{1,0,2}x_1x_3^2 + \binom{3}{1,1,1} x_1(2x_2)x_3 + \binom{3}{0,3,0}(2x_2)^3 + \binom{3}{0,2,1}(2x_2)^2x_3 + \binom{3}{0,1,2}(2x_2)x_3^2 + \binom{3}{0,0,3}x_3^3$
$= x_1^3 +6x_1^2 +3x_1^2x_3 + 12x_1x_2x_3 + 3x_1x_3^2 + 12x_1x_2x_3 +8x_2^3 + 12x_2^2x_3 + 6x_2x_3^2 + x_3^3$
Illustrative examples on multinomial theorem on different digit (method of solution)

Example 1:

$(d_1d_2d_3)^n = (d_1p^2 + d_2p^1 + d_3p^0)^n = (x_1+ x_2 +x_3)^n$
Where $x_1 = d_1p^2$, $x_2 = d_2p^1$, $x_3 = d_3p^0$. Also d_1, d_2, d_3 are positive integers and p=10
Multinomial Expansion of the form $(d_1d_2d_3)^n$, where values of variables are given.

Example 2:

Expanding $(123)^2$ using the multinomial expansion is given by;
$(123)^2 = (d_1p^2 + d_2p^1 + d_3p^0)^2 = (1p^2 + 2p^1 +3p^0)^2$
The Modified detached coefficients method (method of solution) is illustrated below:
Expansion of $(d_1d_2d_3)^n$ using the Modified Detached Coefficients Method.

Example 3:

Expanding $(123)^2$ using the Modified Detached Coefficient Method is given by ;
$(123)^2 = (d_1p^2 + d_2p^1 + d_3p^0)=(1p^2 + 2p^1 +3p^0)^2$
$\Rightarrow(1p^2 + 2p^1 +3p^0)^2$
$= p^4(1)+p^3(4)+p^2(10)+p^1(12)+p^0(9)$, when p=10.
$= p^4(1)+p^3(4)+p^2(10)+p^1(10+2)+p^0(9)$
$= p^4(1)+p^3(4)+p^2(p)+p^1(p+2)+p^0(9)$
$= 1p^4+4p^3+1p^3+1p^2+2p^1+9p^0$
$=1p^4+5p^3+1p^2+2p^1+9p^0$
Therefore $(123)^2 = 15129$
Expanding $(d_1d_2d_3)^n$ using the Staircase Method.

Example 4:

Expanding $(123)^2$ using the Staircase Method is given by;
$(123)^2 = (d_1p^2 + d_2p^1 + d_3p^0)^2 =(1p^2 + 2p^1 +3p^0)^2$
$\Rightarrow(1p^2 + 2p^1 +3p^0)^2$
$= p^4(1)+p^3(4)+p^2(10)+p^1(12)+p^0(9)$

$(123)^2$	=	09
		+12
		+10
		+04
		+01
$(123)^2$	=	15129

Given a power number $Q = (123)^2$ convert to equivalent multinomial expansion by inserting powers of p = 10 as coefficients of the digits in the number as shown below:

$Q=(123)^2 = (1*10^2+2*10^1+3*10^0)^2$
$=(1*p^2+2*p^1+3*p^0)^2$, where p=10
In general if x_1, x_2,x_3, … x_m are order digits of the number Q then $Q =(x_1x_2x_3…x_m)^n$ is the required number, where n is a positive integer.
$\Rightarrow Q= (x_1p^{m-1}+x_2p^{m-2}+…+x_mp^0)^n$
$=(y_1+y_2+…+y_m)^n$, where $y_1 = x_1p^{m-1}$, $y_2 = x_2p^{m-2}$, $y_3 = x_mp^0$

Step 0:

Convert $(x_1x_2x_3…x_m)^n$ to Multinomial form, i.e.,
$Q = (x_1p^{m-1}+x_2p^{m-2}+…+x_mp^0)^n$
$=(y_1+y_2+…+y_m)^n$, where $y_t = x_tp^{m-t}$
From the Multinomial Summation;
$Q = (y_1+y_2+…+y_m)^n$ is expressed as

$$(y_1+y_2+…+y_m)^n = \sum_{k_1+k_2+\cdots+k_m=n} \binom{n}{k_1,k_2,k_3} y_1^{k_1}y_2^{k_2}\ldots y_m^{k_m}$$

Results and discussion

Step 1:
Expand the multinomial using the staircase multinomial expansion, i.e.

$$(p^{m-t}x_1+p^1x_2+…+p^0x_m)^n = \sum_{k_1+k_2+\cdots+k_m} \binom{n}{k_1,k_2,k_m} \prod_{t=1}^{m} x_t^{k_t}\, p^{(m-t)k_t}$$

Where $1\le t\le m$, P=terms determinant
x_1, x_2 … x_m are positive integers.

$$\binom{n}{k_1,k_2,\ldots,k_m} = \frac{n!}{k_1!\,k_2!\ldots k_m!}$$

Step 2:

Input the values of x_1, x_2, …, x_m into the expanded form.

Step 3:

Substitute 10=p in the expanded form $\sum_{i=0}^{n} a_i p^i$, if all a_i's are single digits then extract coefficients as results otherwise go to step 5 if not go to step 7. Where a_1, a_2, …, a_n are coefficients of the p's and p = 10.

Step 4:

Write coefficients with digits more than 1 as placeholder in the expansion of p.

Step 5:

Terms of the same powers are added together.

Step 6:

Rearrange terms in descending powers of p. All coefficients in the current expansion will be of single digit.

Step 7:

Extract the coefficients of the p's as the result of the power number. Given a power number Q = $(123)^2$ convert to equivalent multinomial expansion by inserting powers of p = 10 as coefficients of the digits in the number as shown below:

$Q=(123)^2 = (1*10^2+2*10^1+3*10^0)^2$
$=(1*p^2+2*p^1+3*p^0)^2$, where p=10

In general if x_1, x_2,x_3,x_m are order digits of the number Q then Q $=(x_1x_2x_3\ldots x_m)^n$ is the required number, where n is a positive integer.

$\Rightarrow Q= (x_1p^{m-1}+x_2p^{m-2}+\ldots+x_mp^0)^n$
$=(y_1+y_2+\ldots+y_m)^n$,where $y_1 = x_1p^{m-1}$, $y_2 = x_2p^{m-2}$, $y_3 = x_mp^0$

Algorithms involved in staircase method

Step 0:

Convert $(x_1x_2x_3\ldots x_m)^n$ to Multinomial form, i.e.,
$Q = (x_1p^{m-1}+x_2p^{m-2}+\ldots+x_mp^0)^n$
$=(y_1+y_2+\ldots+y_m)^n$, where $y_t = x_tp^{m-t}$
From the Multinomial Summation;
$Q = (y_1+y_2+\ldots+y_m)^n$ is expressed as

$(y_1+y_2+\ldots+y_m)^n$

$$= \sum_{k_1+k_2+\cdots+k_m=n} \binom{n}{k_1,k_2,k_3} y_1^{k_1} y_2^{k_2} \ldots y_m^{k_m}$$

Step 1:

Expand the multinomial using the staircase multinomial expansion, i.e.
$(p^{m-t}x_1+p^1x_2+\ldots+p^0x_m)^n$

$$= \sum_{k_1+k_2+\cdots+k_m} \binom{n}{k_1,k_2,k_m} \prod_{t=1}^{m} x_t^{k_t}\, p^{(m-t)k_t}$$

Where $1\le t\le m$, p=terms determinant
x_1, x_2, …., x_m are positive integers.

$$\binom{n}{k_1,k_2,\ldots,k_m} = \frac{n!}{k_1!\,k_2!\ldots k_m!}$$

Step 2:

Input the values of $x_1,x_2,\ldots,x_m$ into the expanded form.

Step 3:

The results of the terms are now arranged in the staircase form starting with the term that contains p^0 followed by $p^1,p^2,\ldots,p^m$.

Step 4:

Add staircase digit wise, one digit to the left and placing them successively beneath the previous coefficient.

Computing the cubes of integers, using the coefficients

Problem: Evaluate Q=(x1x2x3)3 using the Modified Detached Coefficients method.

Step 0:

Convert Q to a multinomial, $Q=(p^2x_1+p^1x_2+p^0x_3)^3$

Step 1:

Use the staircase multinomial expansion to expand $(p^2x_1+p^1x_2+p^0x_3)^3$

$Q=(p^2x_1+p^1x_2+p^0x_3)^3$
$=p^6(x_1^3)+p^5(3x_1^2x_2)+p^4(3x_1x_2^2+3x_1^2x_3)+p^3(x_2^3+6x_1x_2x_3)+p^2(3x_2^2x_3+3x_1x_3^2)+p^1(3x_2x_3^2)+p^0(x_3^3)$ …..(1)

Step 2:

Input the values of x_1,x_2,x_3 into eq. (1) in step (1)

Step 3:

Write coefficients with digits more than 1 as placeholder in the expansion of p

Step 4:

Terms of the same powers are added together.

Step 5:

Rearrange terms in descending powers of p. All coefficients in the current expansion will be of single digit.

Step 6:

Extract the coefficients of the p's as the result of the power number.

Example 5:

Computing $Q=(123)^3$,Using the coefficients method.
Solution steps:

Step 0:

$Q = (123)^3 =(p^2x_1+px_2+p^0x_3)^3$

Step 1:

Use the staircase multinomial expansion to expand $(p^2x_1+p^1x_2+p^0x_3)^3$

$(p^2x_1+p^1x_2+p^0x_3)^3 =$
$p^6(x_1^3)+p^5(3x_1^2x_2)+p^4(3x_1x_2^2+3x_1^2x_3)+p^2(3x_2^2x_3+3x_1x_3^2)+p^1(3x_2x_3^2)+p^0(x_3^3)$ ….(2)

Step 2: Input the values of x_1=1, x_2 = 2, x_3 = 3 in eq. (2)

$(123)^3$

$=(1p^2+2p+3p^0)^3=p^6(1)+p^5(6)+p^4(21)+p^3(44)+p^2(63)+p^1(54)+p^0(27)$, when p = 10.

$= p^6(1)+p^5(6)+p^4[2(10)+1]+p^3[4(10)+4]+p^2[6(10)+3]+p^1[5(10)+4]+p^0[2(10)+7]$

$= p^6(1)+p^5(6)+p^4[2(p)+1]+p^3[4(p)+4]+p^2[6(p)+3]+p^1[5(p)+4]+p^0[2(p)+7]$

$= p^6(1)+p^5(6)+2p^5+p^4+4p^4+4p^3+6p^3+3p^2+5p^2+4p^1+2p^1+7p^0$

$= 1p^6+8p^5+5p^4+10p^3+8p^2+6p^1+7p^0$

$=1p^6+8p^5+5p^4+(p)p^3+8p^2+6p^1+7p^0$

$=1p^6+8p^5+5p^4+1p^4+8p^2+6p^1+7p^0$

$=1p^6+8p^5+6p^4+0p^3+8p^2+6p^1+7p^0$

The appended coefficients are in their order of placement.

Therefore $(123)^3 = 1860867$

Computing the cubes of integers, using the staircase method

Evaluate $Q = (x_1x_2x_3)^3$ by Staircase Method

Step 0:

Convert Q to a multinomial form $Q=(p^2x_1+p^1x_2+p^0x_3)^3$, when p=10

Step 1:

Expand $(p^2x_1+p^1x_2+p^0x_3)^3$ using the multinomial expansion.

$Q=(p^2x_1+p^1x_2+p^0x_3)^3$

$=p^6(x_1^3)+p^5(3x_1^2x_2)+p^4(3x_1x_2^2+3x_1^2x_3)+p^3(x_2^3+6x_1x_2x_3)+p^2(3x_2^2x_3+3x_1x_3^2)+p^1(3x_2x_3^2)+p^0(x_3^3)$

$Q=(x_1x_2x_3)^3$

$=P^6(x_1^3)+p^5(3x_1^2x_2)+p^4(3x_1x_2^2+3x_1^2x_3)+p^3(3x_2^2x_3+3x_1x_3^2)+p^2(3x_2x_3^2)+p^0(x_3^3)$, when p=10. …(3)

Step 2:

Input the values of x_1, x_2, x_3 into eq. (3) in step (1).

Step 3:

Arrange the coefficients of p^{th} powers in the staircase form by starting coefficient of p^0 on the first line placing the coefficient of $p^1,p^2,\ldots,p^m$ with the last digit of p^i listed below the last but one digit of p^{i-1} .

Step 4:

Add staircase numbers arrangement column-wise as done in multiplication of two numbers.

Example 6:

Computing $Q=(123)^3$, using the staircase method

Step 0:

$Q = (123)^3 =(p^2x_1+px_2+p^0x_3)^3$

Step 1:

Use the staircase multinomial expansion to expand$(p^2x_1+p^1x_2+p^0x_3)^3=$ $p^6(x_1^3)+p^5(3x_1^2x_2)+p^4(3x_1x_2^2+3x_1^2x_3)+p^2(3x_2^2x_3+3x_1x_3^2)+p^1(3x_2x_3^2)+p^0(x_3^3)$ …(4)

Step 2:

Input the values of $x_1=1,x_2 = 2,x_3 = 3$into equation(1) in step(1)

$(123)^3$

$=(1p^2+2p+3p^0)^3=p^6(1)+p^5(6)+p^4(21)+p^3(44)+p^2(63)+p^1(54)+p^0(27)$, when p = 10.

```
(123)^3  =      27
               +54
              +63
             +44
            +21
           +06
          +01
------------------------
(123)^3  =   1860867
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Conclusions

The problem of the method of Detached Coefficients by [3] is that when the coefficient(s) of the terms of a binomial /multinomial expansion is more than 1 digit it does not give the result of a power number. The problem would be modeled using the multinomial expansion. The Modified Detached Coefficients Method and the Staircase Method would be used as methods of solution to resolve the problem of [2]. It is concluded that in terms of computing powers of positive integers, Staircase method has lesser steps than the Modified Detached Coefficients method in terms of computation.

Conflicts of interest

Authors declare no conflict of interest.

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